



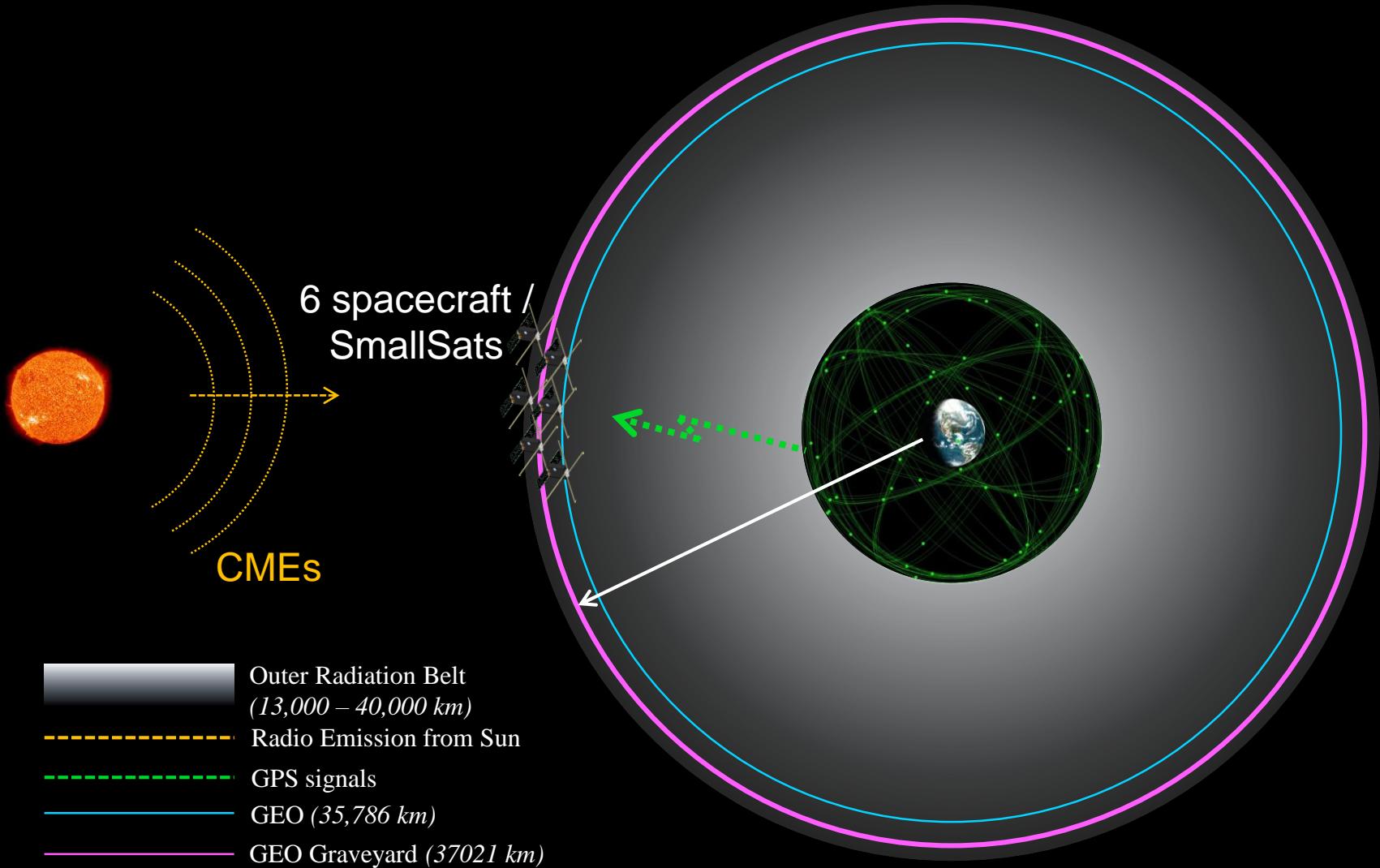
Formation Flying and Position Determination for a Space-Based Interferometer in GEO Graveyard Orbit

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California Institute of Technology

Mission Concept – CME Observer



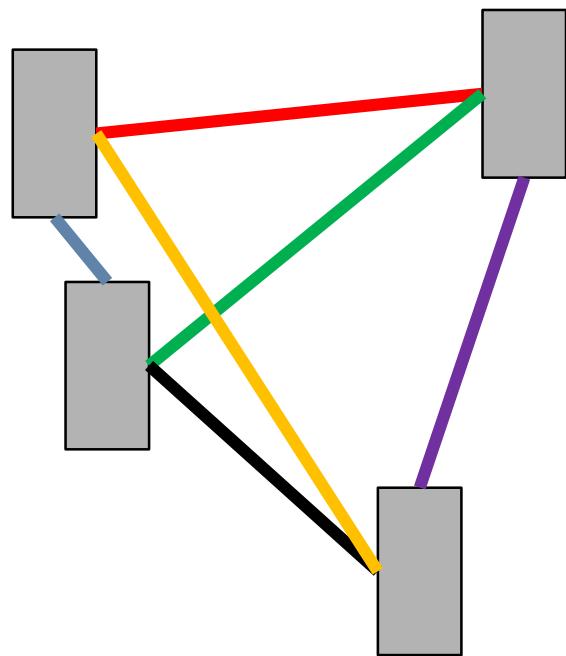
MDNav Drivers

- *Formation Design*
 - Dynamical Systems Theory
 - Visual method
 - Automated method
 - *On-orbit Operations*
 - Deployment
 - Station-keeping
 - *Relative Positioning*
 - GPS enabled
 - UHF cross-linking (in paper)
- 
- Cannot predict CMEs early enough to reconfigure formation, interferometer must be "always on"
- Cold-gas system provides limited ΔV budget; two-week operation cycle
- Target radio wavelengths (0.5-30Mhz) require relative sc/sc position knowledge to <3 meters

Formation Design Strategy

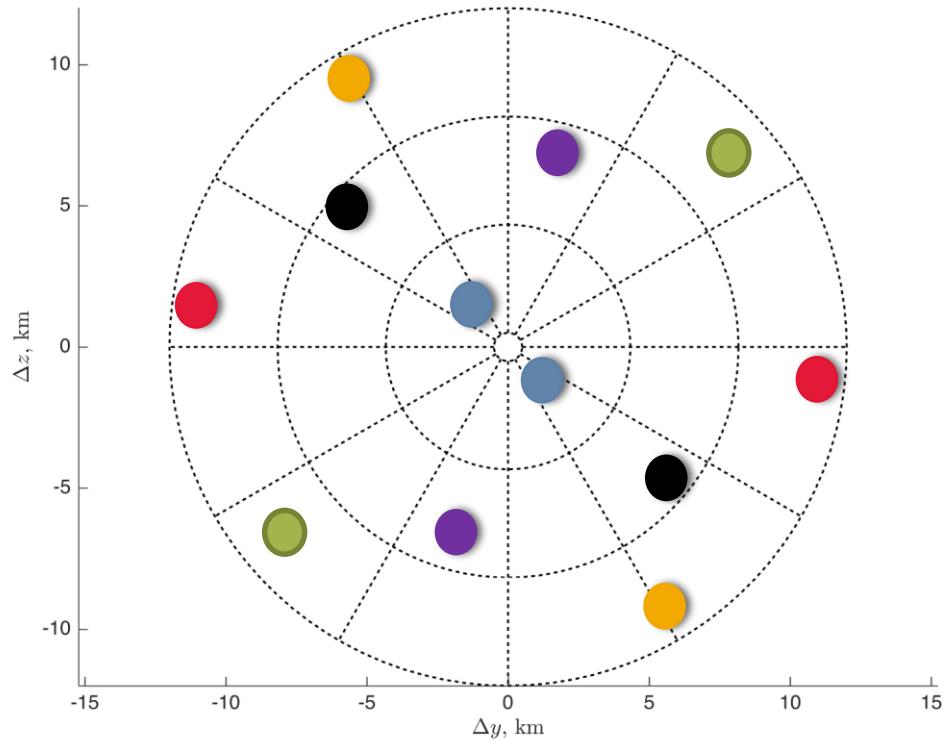
Science Proxy

SC/SC baseline pairs



Euclidean Space:
Normal to Sun line / in plane of sky

Science annulus



Differenced baseline space

Dynamical Systems Theory

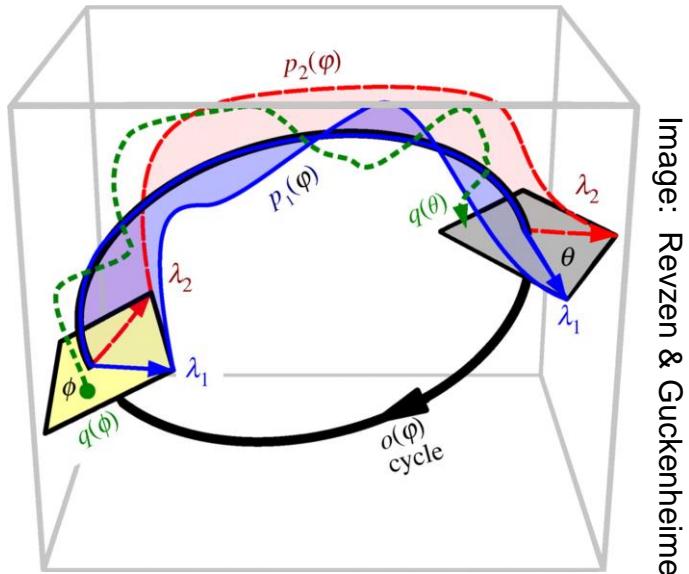


Image: Revzen & Guckenheimer

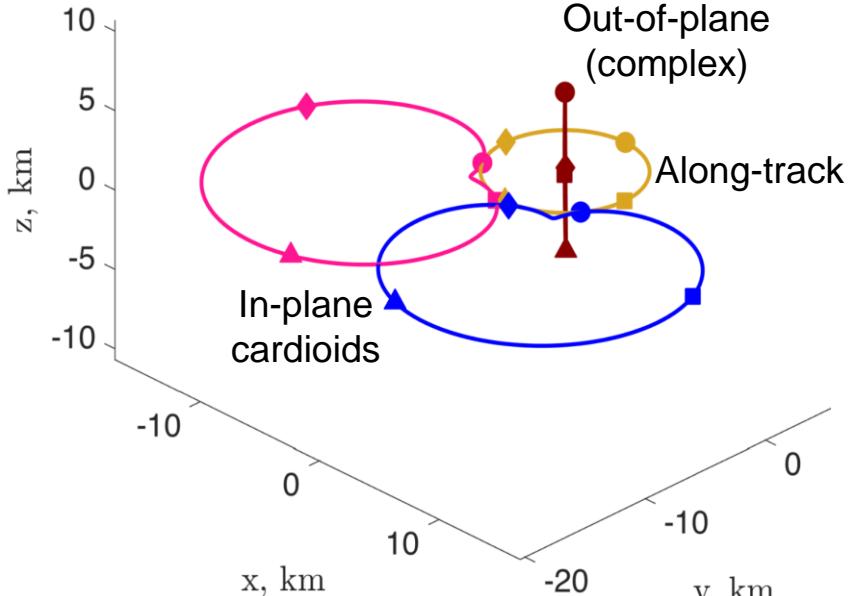
Linear dynamics
relative to periodic orbit $\dot{\vec{x}} = A(t)\vec{x}$

Key Idea: Perturb orbit via linear
combinations of state transition matrix
(STM) eigenvectors*



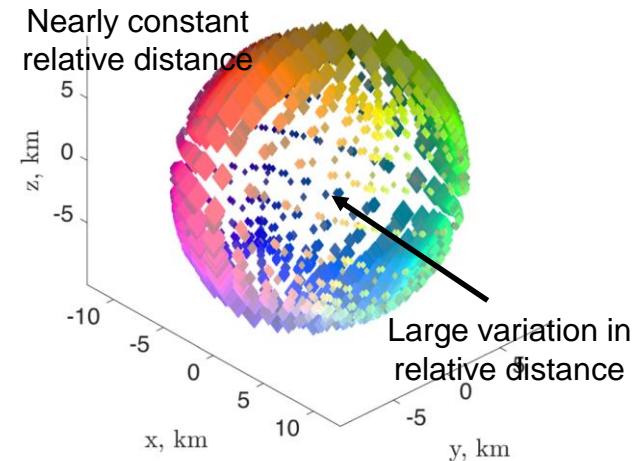
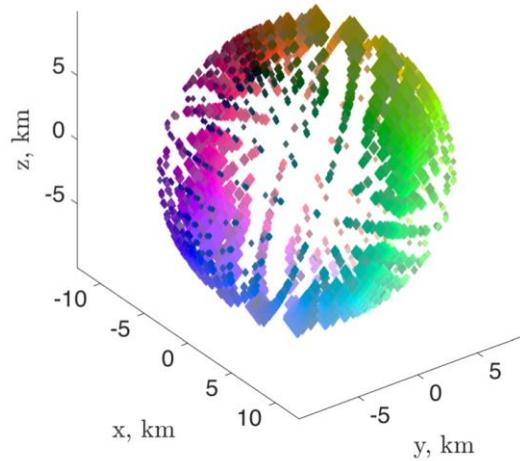
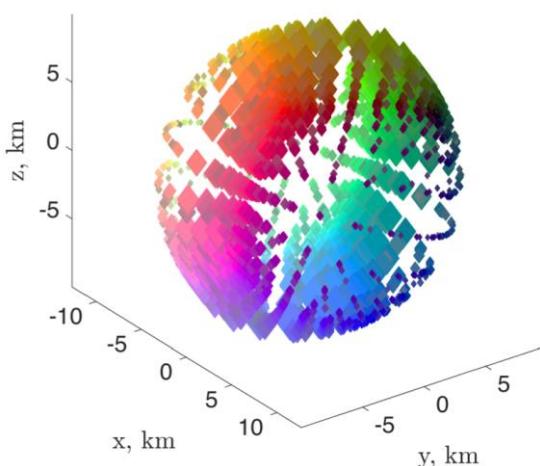
Excite fundamental motions while
preserving orbital period / energy

GEO Graveyard – Relative Motions

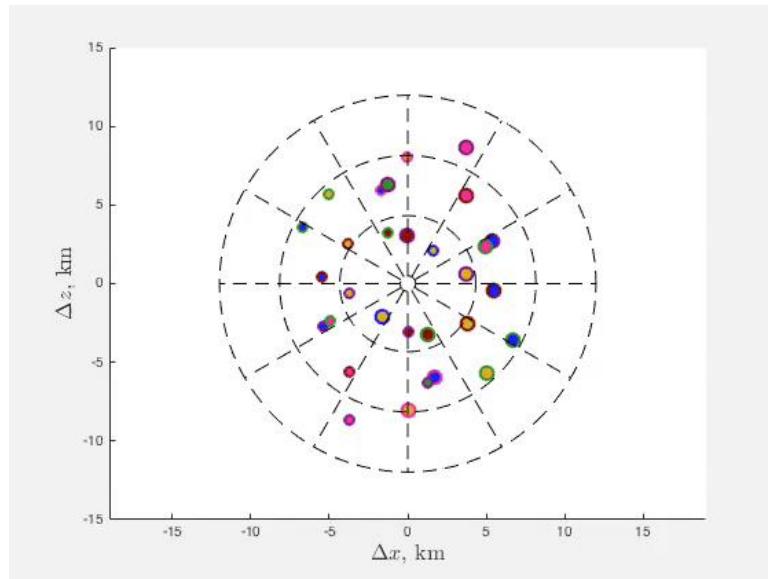
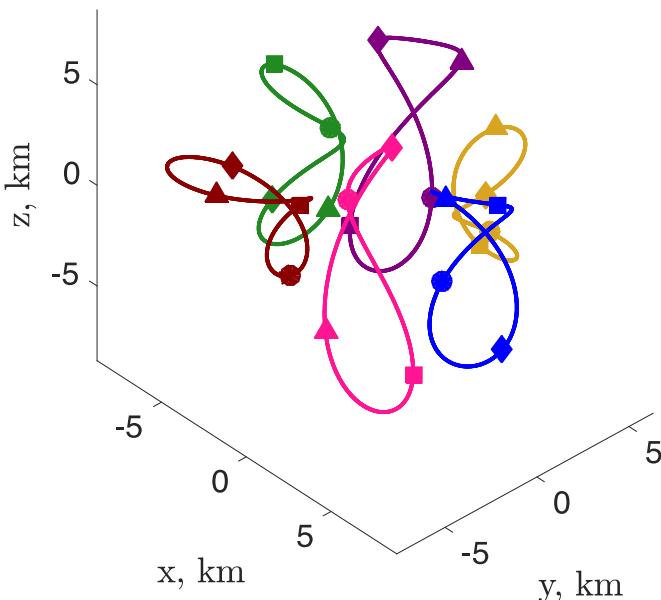


*NOTE: For conic orbits, this method provides equivalent solutions to Clohessy-Wiltshire, etc.

Visual Design Strategy

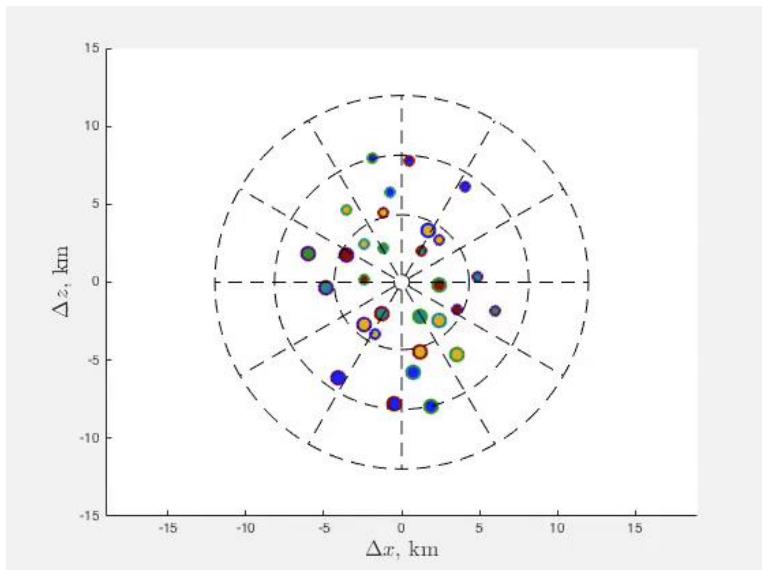
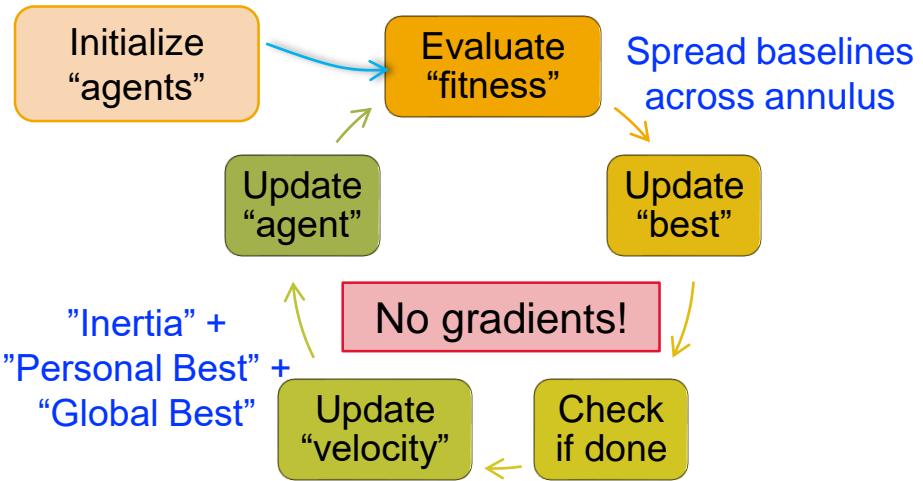
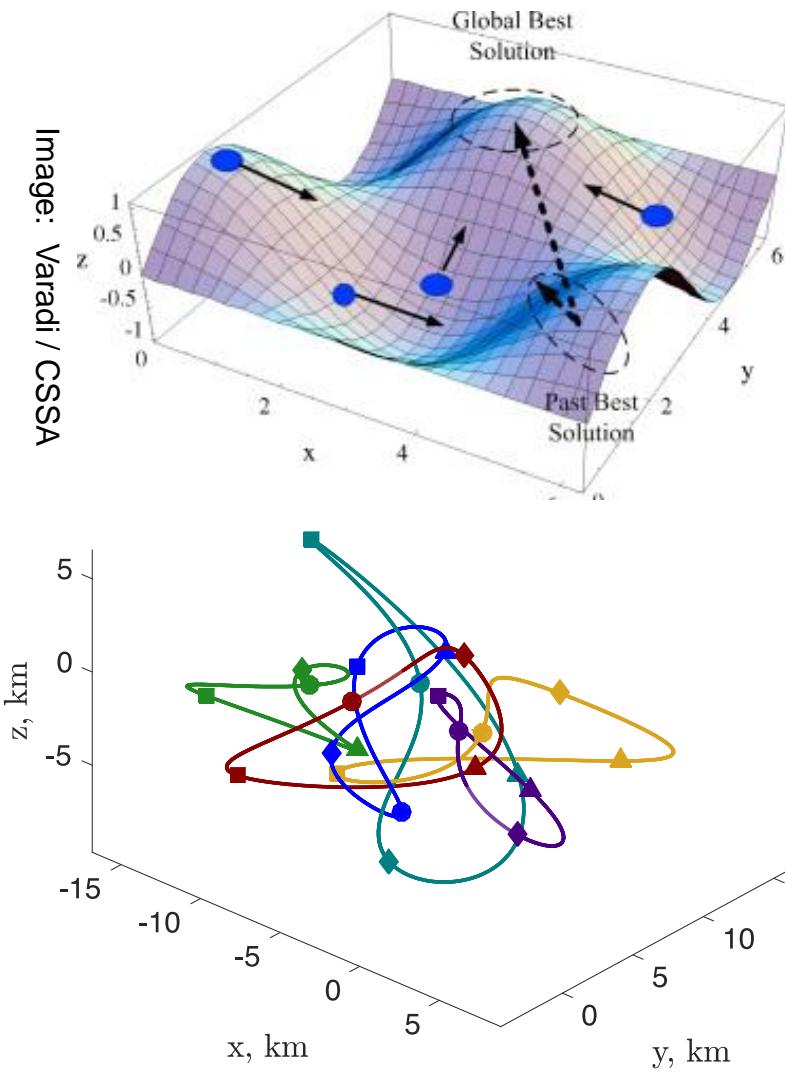


Select perturbed trajectories that remain “close” → 6 S/C, 6 regions



Particle Swarm Optimization

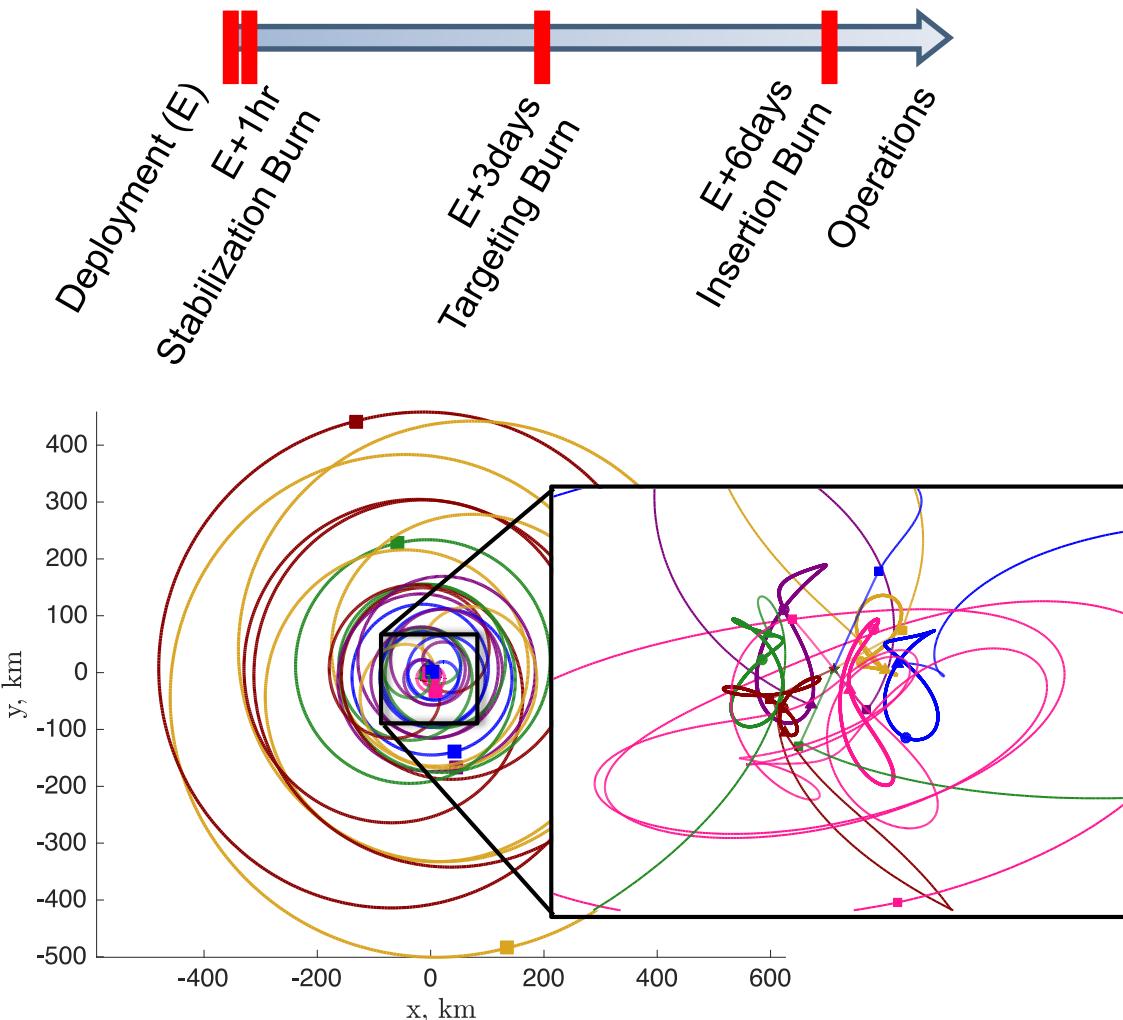
Image: Varadi / CSSA



On-Orbit Operations

Deployment Strategy Monte Carlo

Straight-forward, non-optimized deployment timeline



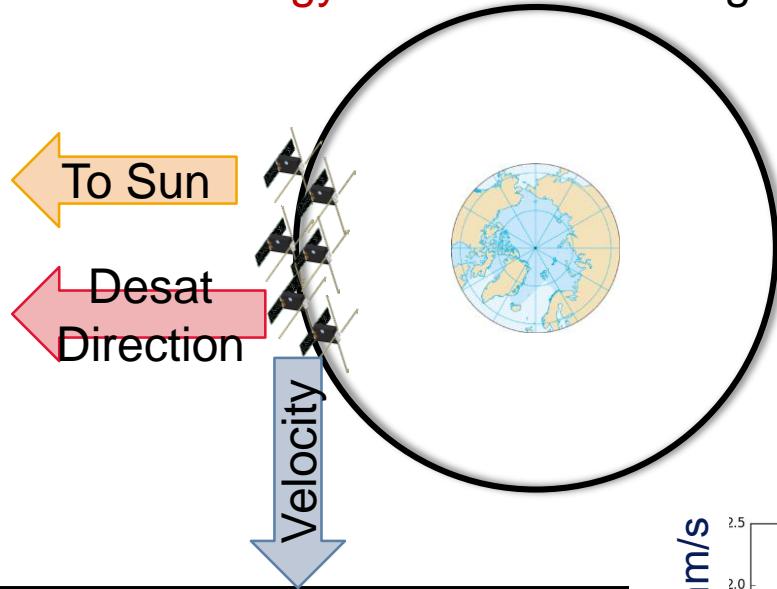
Parameter	Nominal	$1-\sigma$	Units
Deployment Impulse			
Magnitude	1.45	0.13	m/s
Time along Orbit (E)	180	1	min
Direction	*	2	deg
Stabilization Burn			
Magnitude	1.45	0.15	m/s
Coast	60	2	min

Spacecraft	$\Delta V-99$ (m/s)
1 (yellow)	4.5221
2 (blue)	3.4819
3 (pink)	3.9517
4 (red)	4.407
5 (green)	3.5755
6 (purple)	4.0657

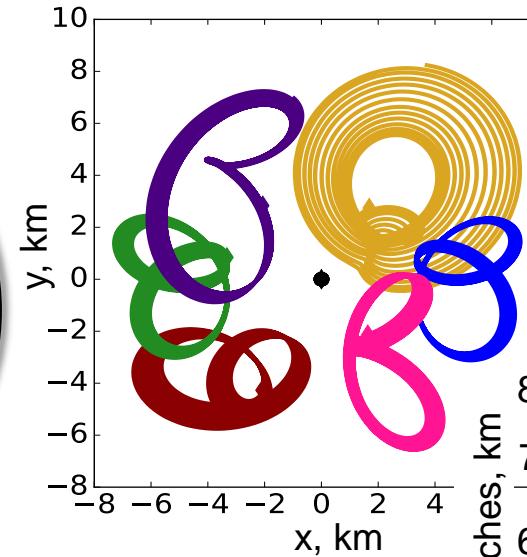
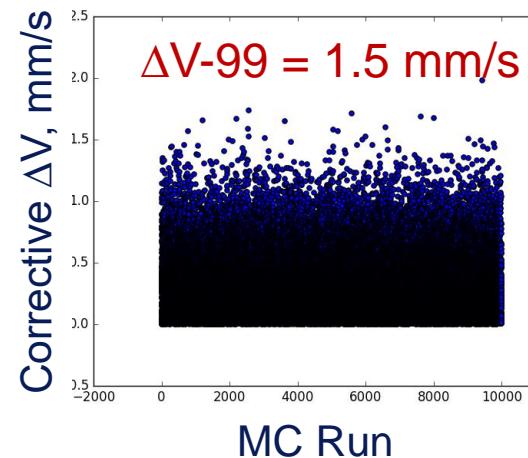
Station-Keeping Monte Carlo

Dominant perturbation → unbalanced momentum wheel desaturations

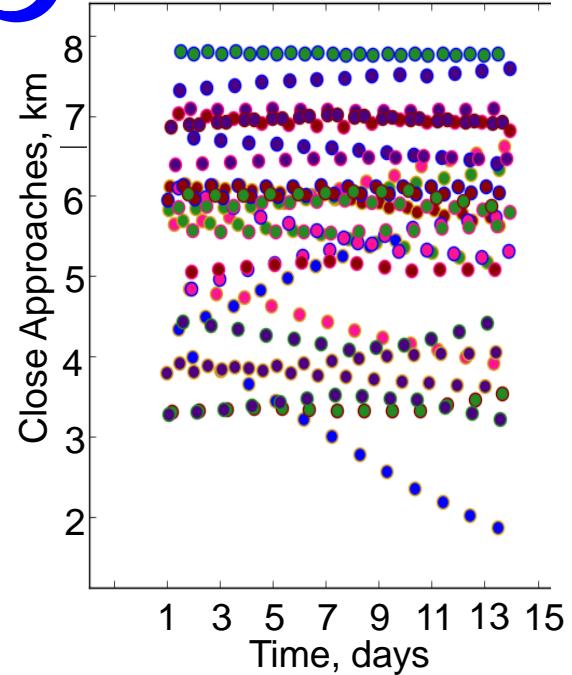
Desat strategy: a matter of timing



Parameter	Value	Units
Nominal Magnitude	3.2	cm/s
Magnitude 1- σ	2	%
Pointing 1- σ	0.33	deg
Interval	75	hrs



Sample 2-week cycle

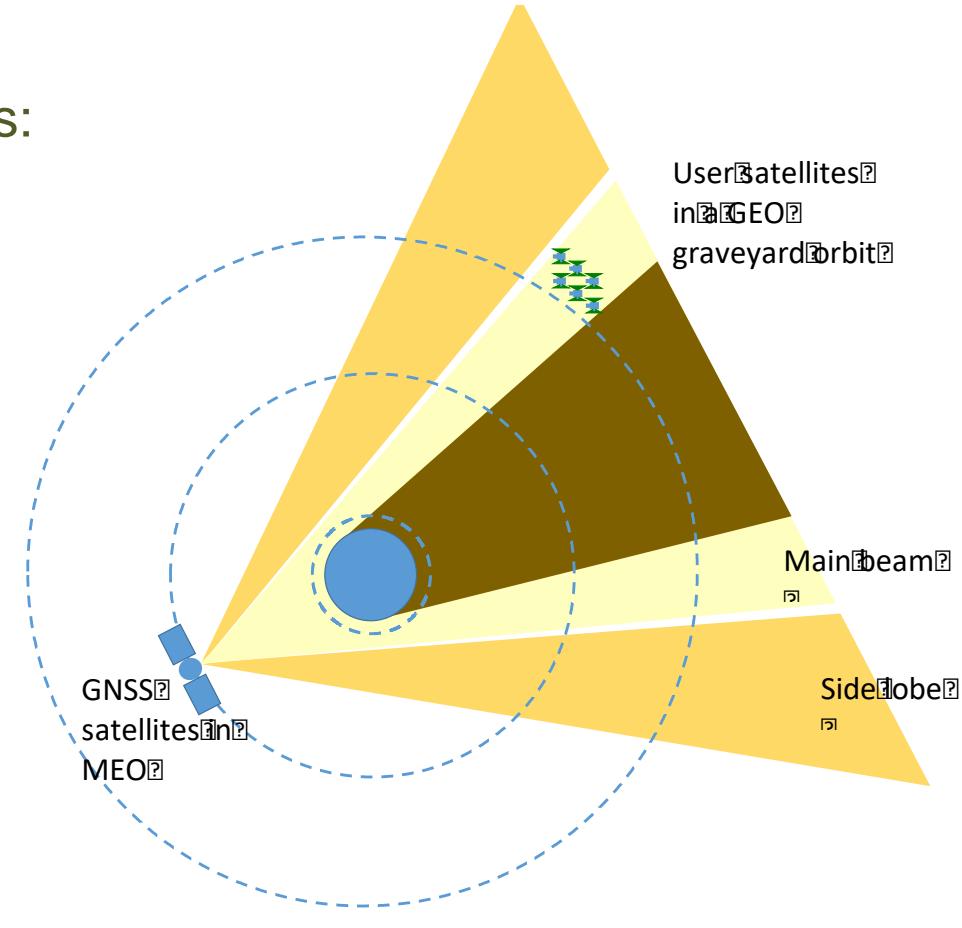


Above-the-Belt GPS Navigation

GPS Navigation Concept

Relative Positioning to 3 meters:

- Estimate individual spacecraft states, compute relative state
- Many perturbations & measurement errors will be common (e.g., path delays)
- Ground processing satisfies science goals, no need for on-orbit knowledge

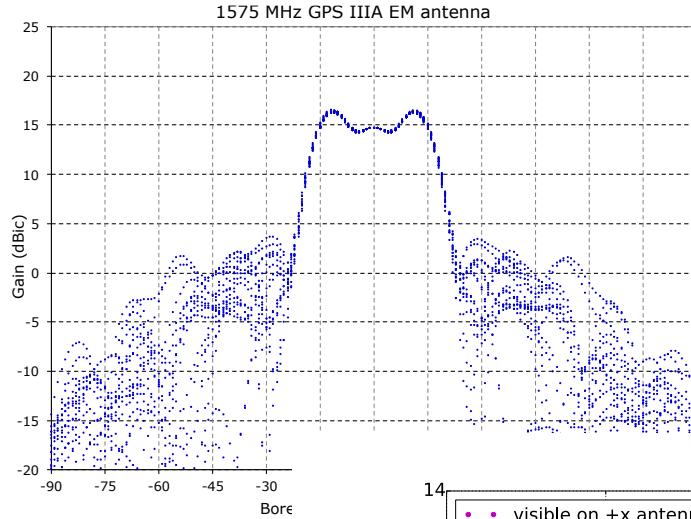


Above the belt GPS navigation heritage:

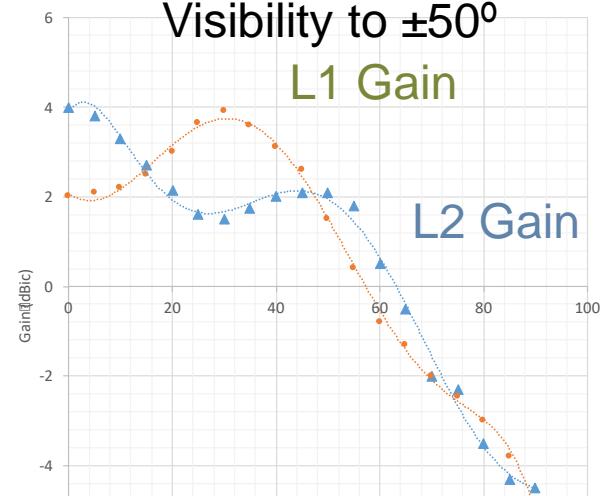
- Magnetospheric Multiscale (MMS) mission [launch March 2015]
- NASA/NOAA GOES-R satellites [first launch Nov. 2016]

Antenna Gain & Visibility

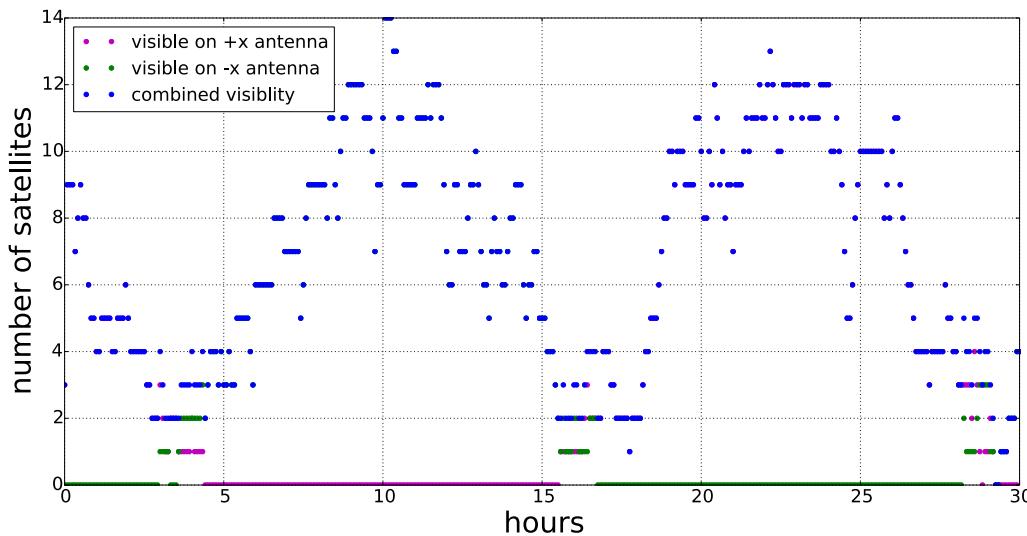
GPS Signal L1 Gain Pattern (L2 similar)
Main lobes extend to $\pm 15^\circ$



GNSS Receiver Gain Pattern
Visibility to $\pm 50^\circ$

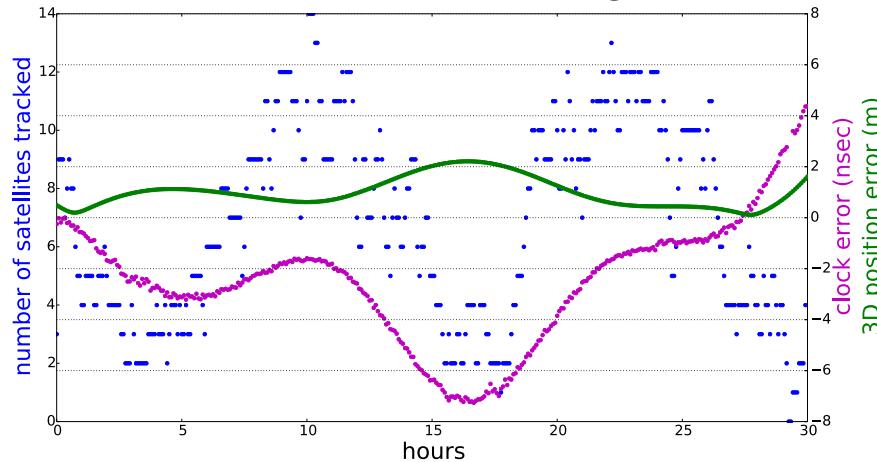


GNSS (GPS +
GLONASS) Satellites
in View
*Mean: 8.6 above
acquisition cut-off*

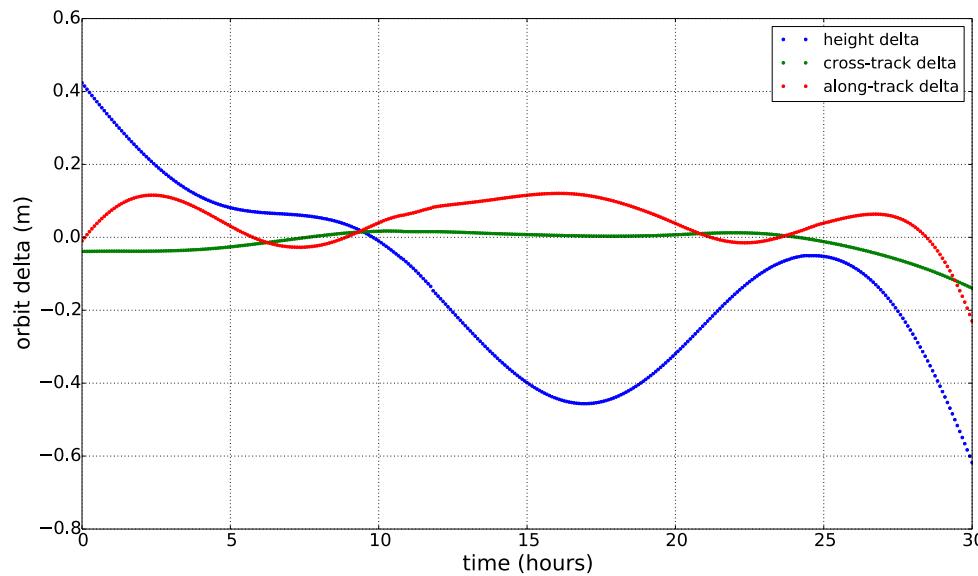
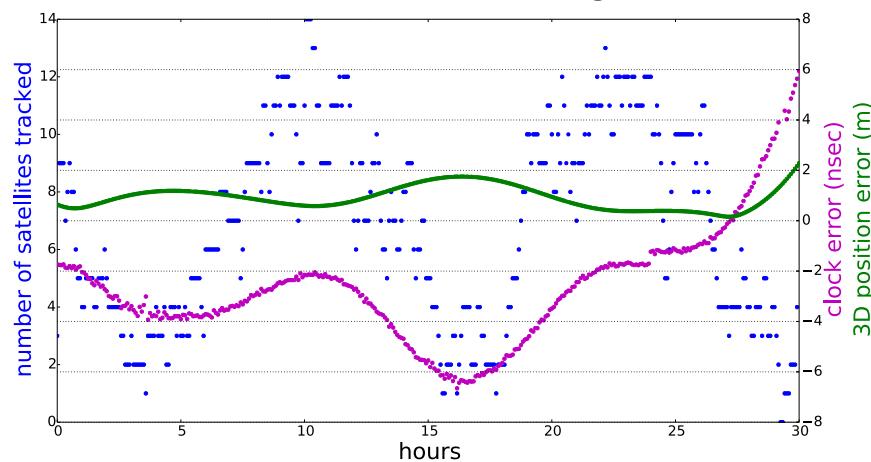


Sample Results – 30-hour Simulation

Absolute Positioning - 1



Absolute Positioning - 3

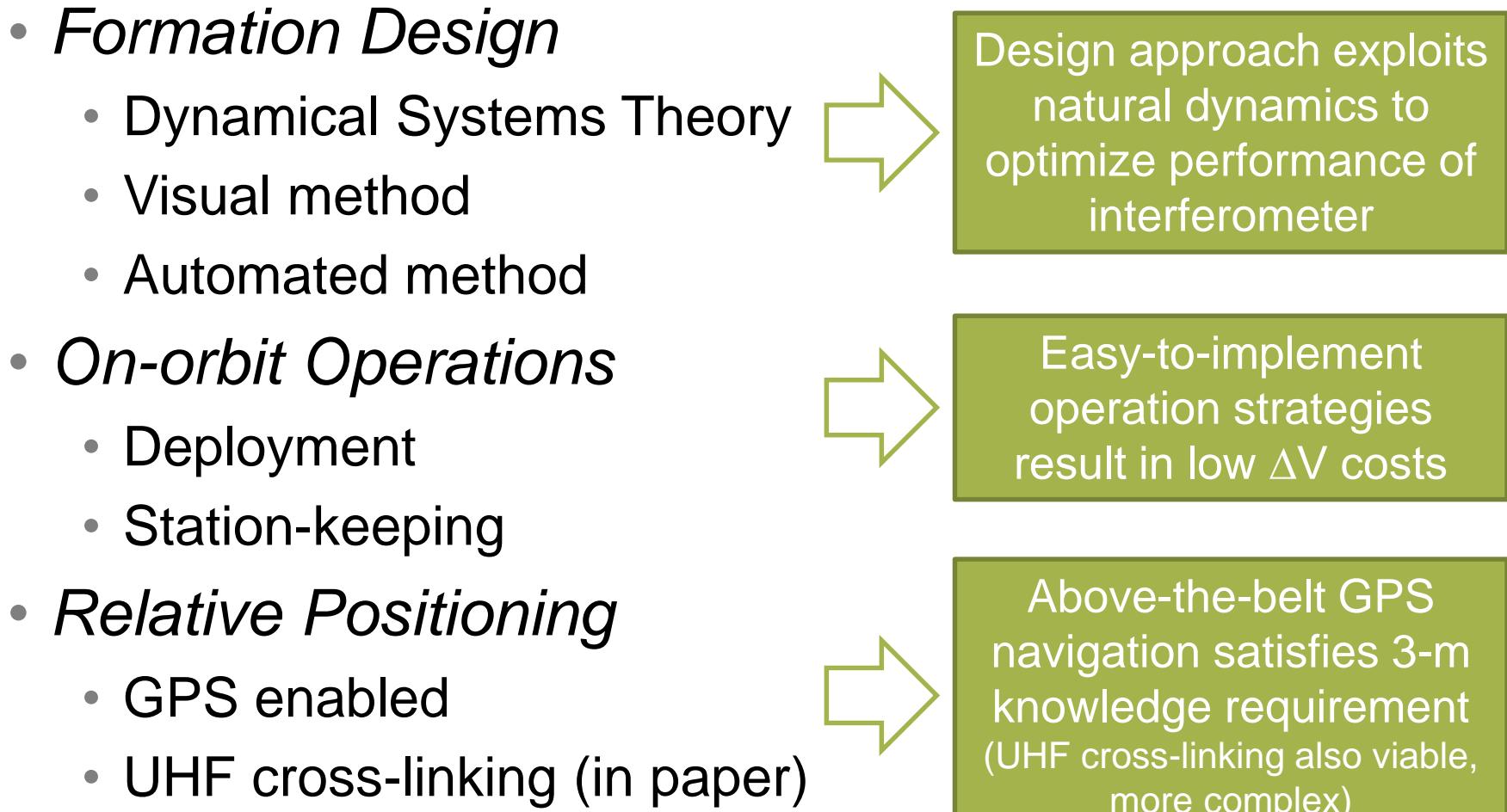


1-to-3 Relative Positioning
3-D RMS Error – 0.51 m

Mean across all sc pairs: 1.08 m

Requirement: 3 m

Summary

- *Formation Design*
 - Dynamical Systems Theory
 - Visual method
 - Automated method
 - *On-orbit Operations*
 - Deployment
 - Station-keeping
 - *Relative Positioning*
 - GPS enabled
 - UHF cross-linking (in paper)
- 
-
- Design approach exploits natural dynamics to optimize performance of interferometer
-
- Easy-to-implement operation strategies result in low ΔV costs
-
- Above-the-belt GPS navigation satisfies 3-m knowledge requirement (UHF cross-linking also viable, more complex)



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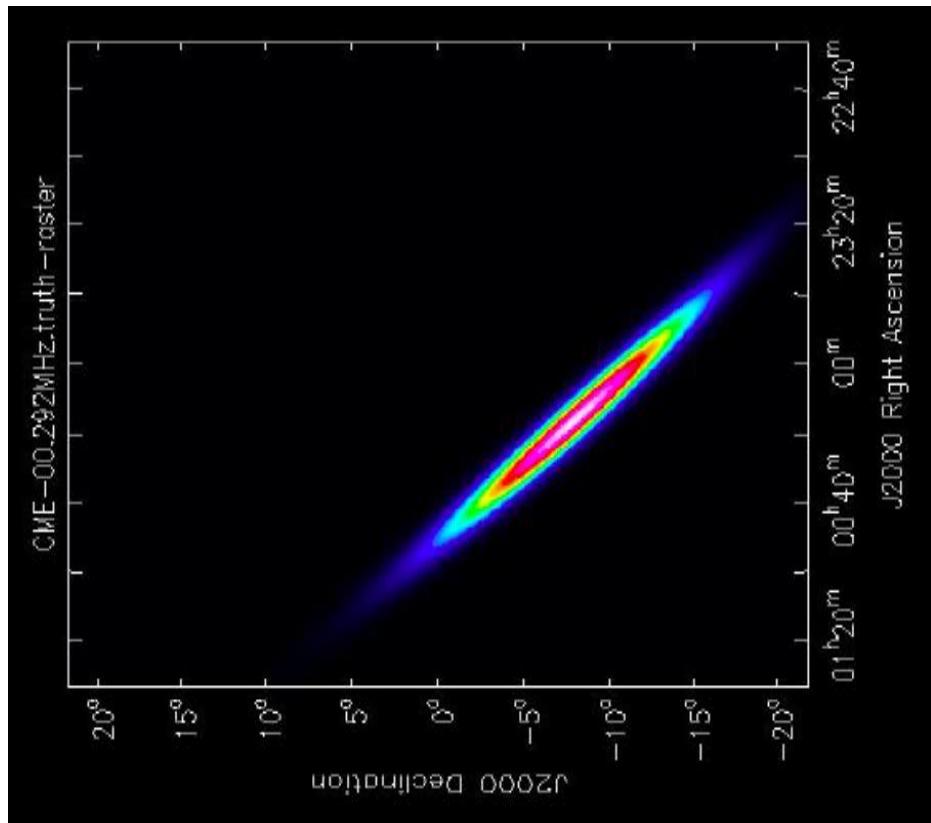
Mission Concept

- **Goal:** characterize behavior & evolution of Coronal Mass Ejections (CMEs) and Solar Energetic Particle (SEP) events
- **Method:** measure frequencies typically cut-off by ionosphere -> formation of spacecraft -> space-based VLBI

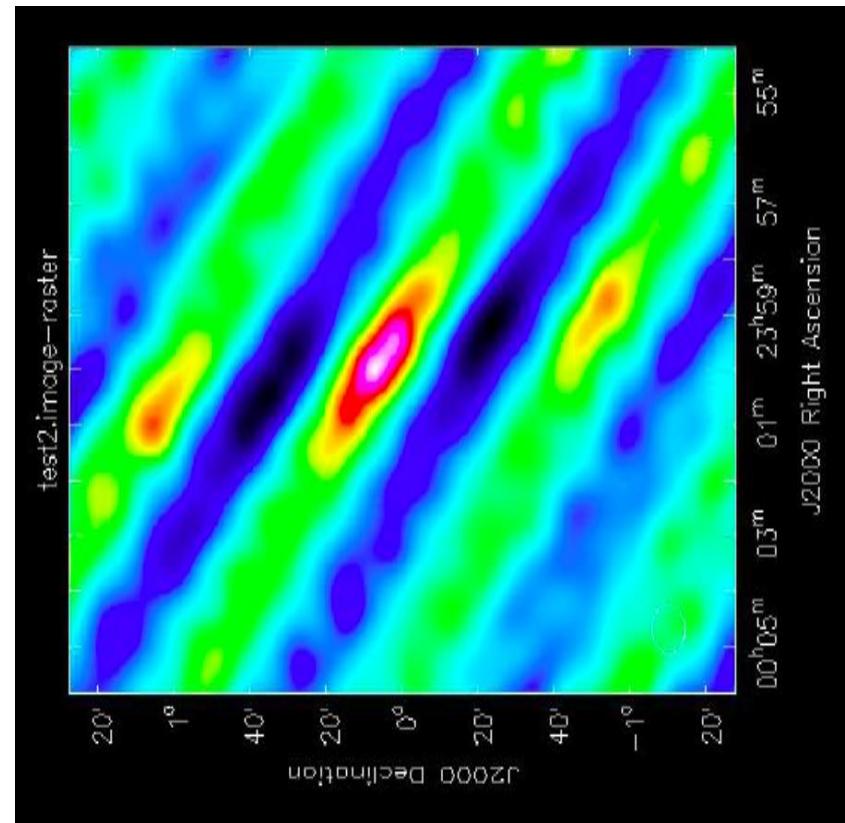


Desired Radio Images

“Truth”



“Reconstruction”



Note that “reconstructed” image is from a different case than “truth” image

Classical Formation Design

Clohessy-Wiltshire

- Relative, linearized EOMs
 - Non-linear extensions
- Chief in circular orbit
 - Elliptical extensions
- Used often for rendezvous

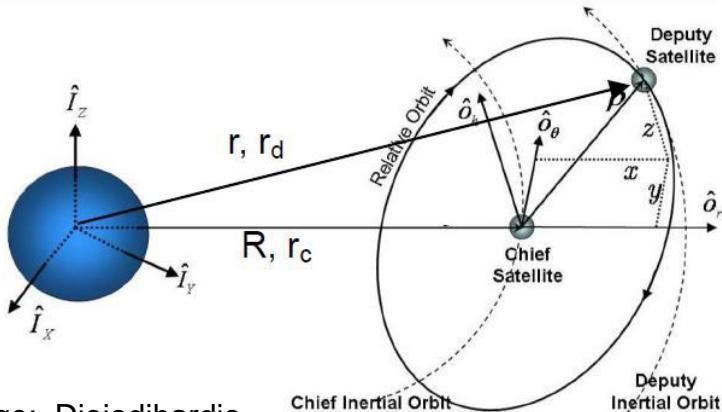


Image: Djojodihardjo

Varied Periodic Orbits

- Body-centered, full motion
 - Multi-body regimes as well
- Fewer orbital assumptions
- Useful for “spread” constellations

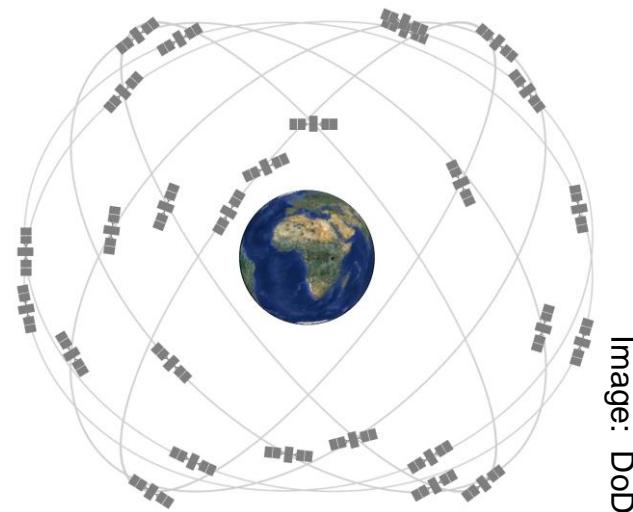


Image: DoD

Initializing Relative Motion

- Different Analysis Methods
 - Clohessy-Wiltshire
 - Varied Conic Elements
 - Dynamical Systems Theory*

Linear, periodic system $\dot{\vec{x}} = A(t)\vec{x}$

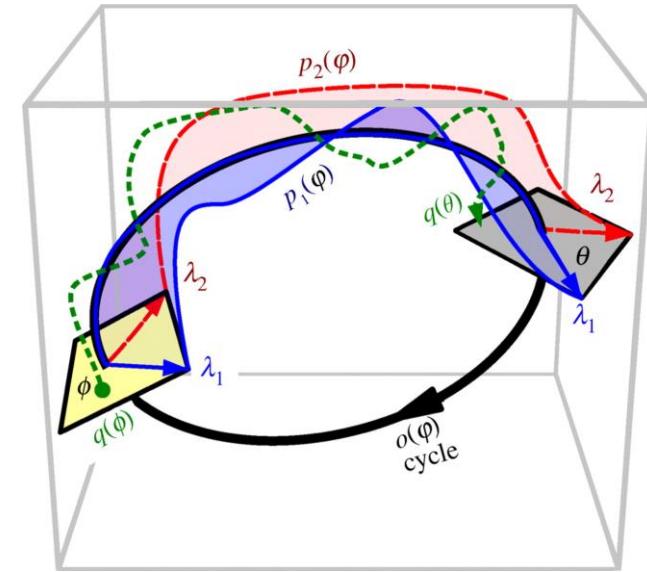
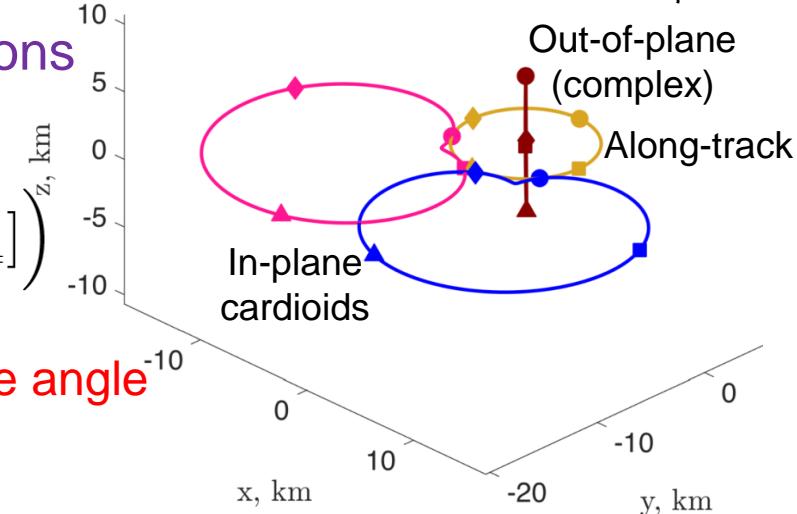


Image: Revzen & Guckenheimer

GEO Graveyard - Perturbing Initial Conditions

$$u(\varepsilon, \theta) = \boxed{\varepsilon_1} \hat{\Gamma}_1 + \boxed{\varepsilon_2} \hat{\Gamma}_2 + \boxed{\varepsilon_3} \hat{\Gamma}_3 + \boxed{\varepsilon_4} \left(\cos(\theta_4) \Re[\hat{\Gamma}_4] - \sin(\theta_4) \Im[\hat{\Gamma}_4] \right)$$



For each S/C, select 4 magnitudes & 1 phase angle

- Energy / period is *preserved* by step

Eigenvectors & Manifolds

Monodromy matrix eigenvalues & eigenvectors

$$M = \phi^{-1}(0)\phi(T) \rightarrow \gamma_i, \hat{\Gamma}_i$$

Single eigenvalue manifold step (from Z. Olikara's 2010 Master's Thesis)

$$\vec{u}(\tau, \theta, \varepsilon) = \varepsilon \left(\cos(\theta) \underbrace{\Re[\hat{\Gamma}(\tau)]}_{\text{(un)stable manifold}} - \sin(\theta) \Im[\hat{\Gamma}(\tau)] \right)$$

Linear combination manifold step

$$\vec{u}(\tau, \theta_i, \varepsilon_i) = \sum_{i=1}^n \varepsilon_i \left(\cos(\theta_i) \Re[\hat{\Gamma}_i(\tau)] - \sin(\theta_i) \Im[\hat{\Gamma}_i(\tau)] \right)$$

Baseline GEO Graveyard



GEO Graveyard / Super GEO

Altitude: 37,021 km

GEO Altitude: 35,786 km

Period: 25 hrs

Eigenvectors:

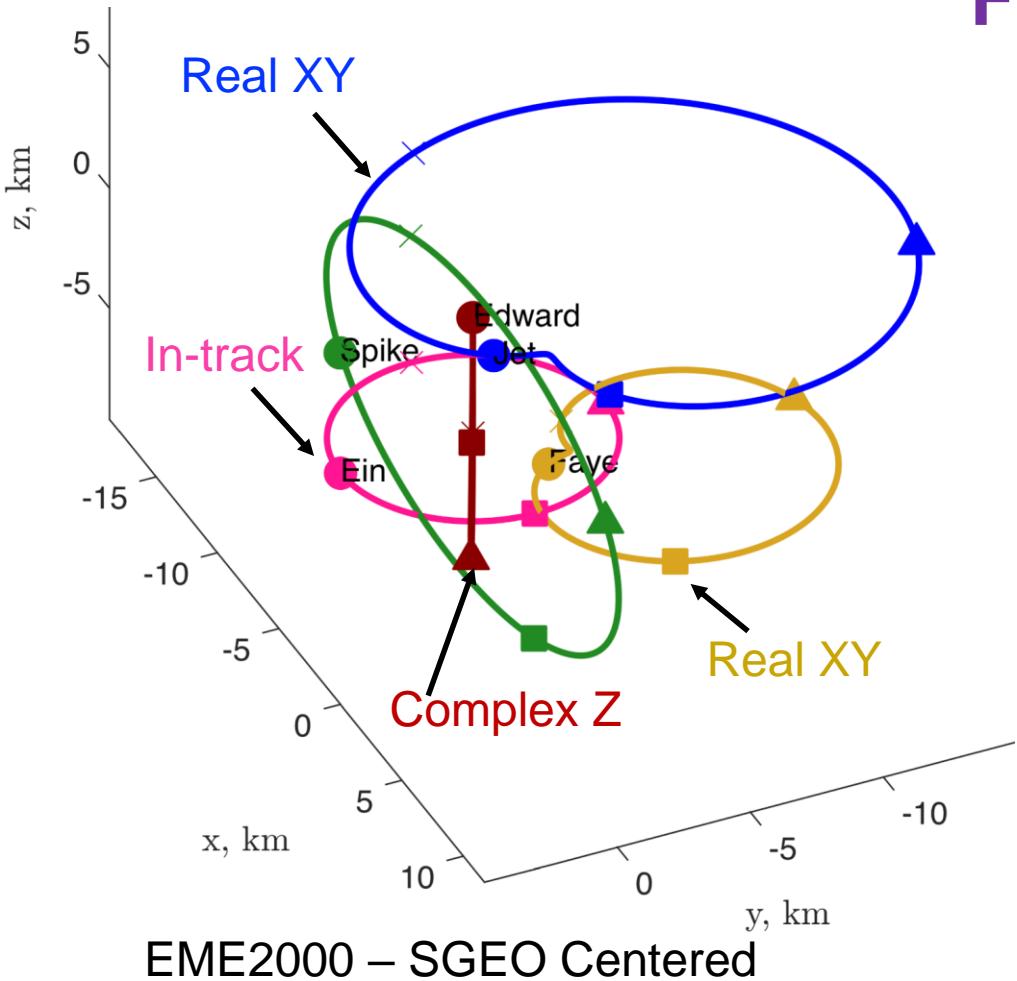
1 along track (real)

1 “center” (complex)

2 “others” (real)

$$\begin{aligned}\vec{u}(\tau) &= \varepsilon_1 \hat{\Gamma}_1(\tau) + \varepsilon_2 \hat{\Gamma}_2(\tau) + \varepsilon_3 \hat{\Gamma}_3(\tau) & \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 &\rightarrow \pm \\ &+ \varepsilon_4 \left(\cos(\theta_4) \Re[\hat{\Gamma}_4(\tau)] - \sin(\theta_4) \Im[\hat{\Gamma}_4(\tau)] \right) & \theta_4 &\rightarrow [0^\circ, 180^\circ]\end{aligned}$$

SGEO – Individual Eigenvectors



For one s/c

$\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \rightarrow \pm 5 \text{ km}$

$\theta_4 \rightarrow [0^\circ, 180^\circ]$

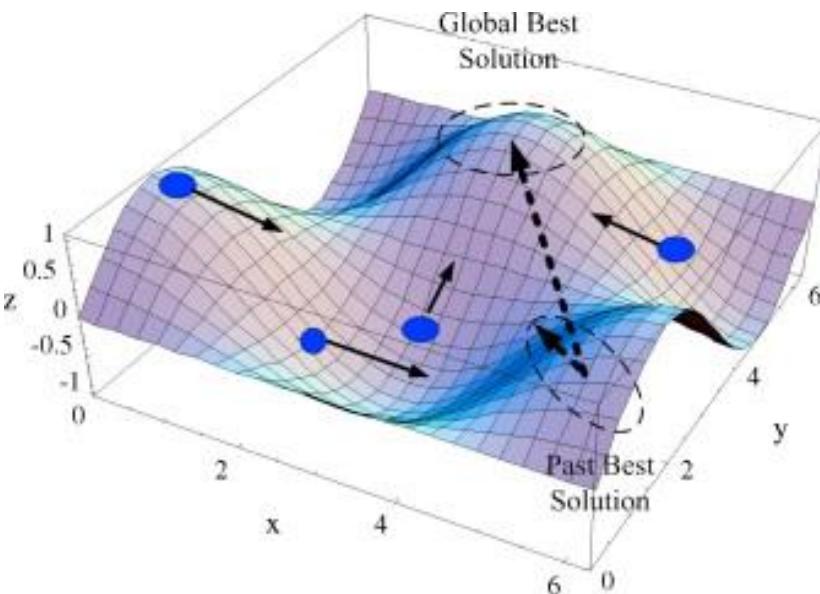
0.1 km steps, 1° steps

1.9e10 combinations!

Large search space
w/ many local
optima

Particle Swarm Optimization

Image: Varadi / CSSA



Optimization variables → ±
 $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ → ±
 $\theta_4 \rightarrow [0^\circ, 180^\circ]$

$$\vec{v}_i = w_I \vec{v}_i + w_P \varpi (\vec{p}_i - \vec{x}_i) + w_G \varpi (\vec{g}_i - \vec{x}_i)$$

Inertia "Agent" best Global best

